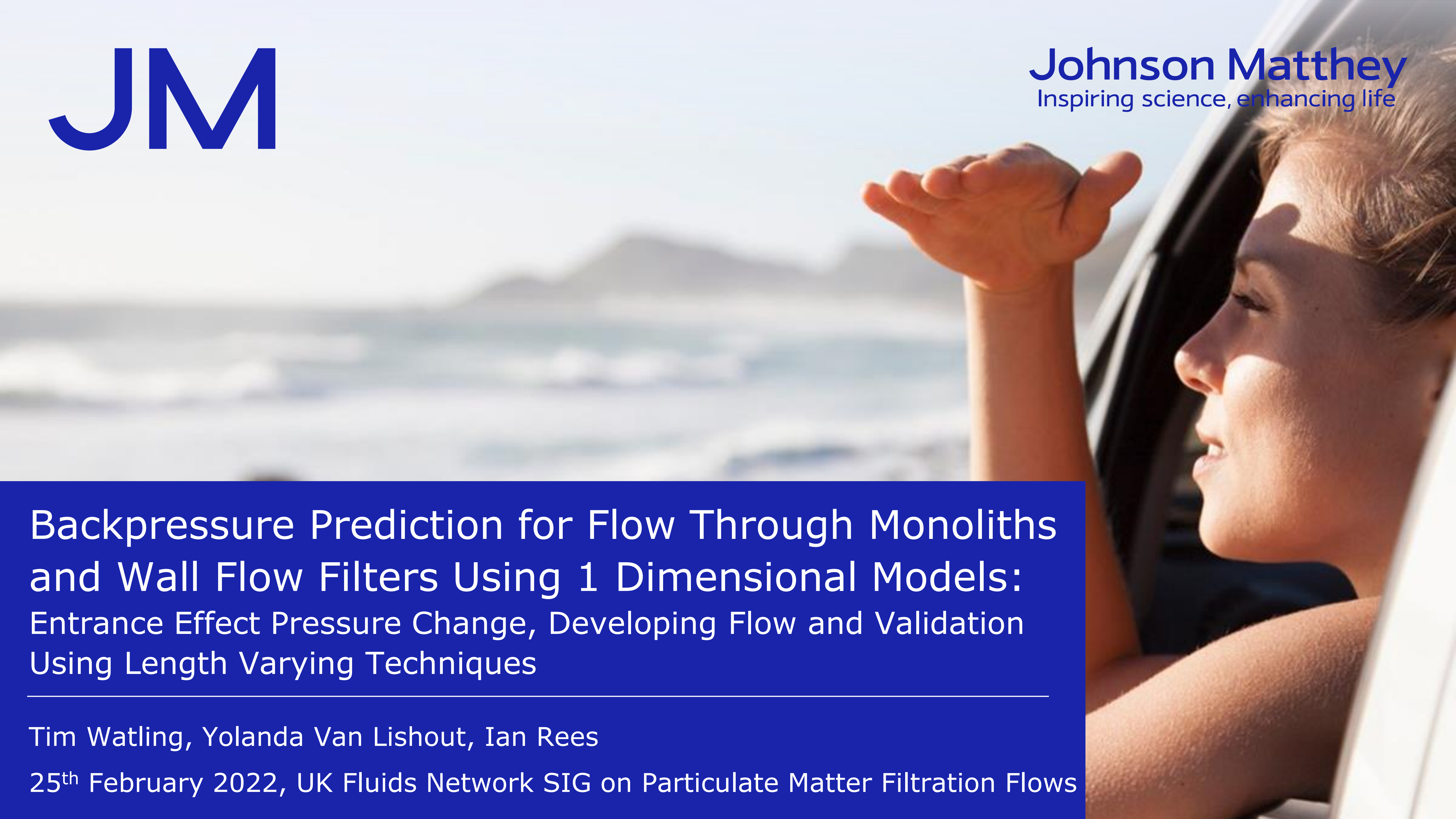
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Backpressure Prediction for Flow Through Monoliths and Wall Flow Filters Using 1 Dimensional Models:

Entrance Effect Pressure Change, Developing Flow and Validation Using Length Varying Techniques

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25th February 2022, UK Fluids Network SIG on Particulate Matter Filtration Flows

Objective

Open Questions on Backpressure Prediction for Monoliths and Particulate Filters

- Aim to address a number of open questions on backpressure prediction for Flow-Through Monoliths (FTM) and Particulate Filters (PF) using 1-D models:
  - No agreement on equation for pressure drop due to flow contraction at FTM or PF entrance
  - How should friction factor correlations for developing flow in FTM be combined with an equation for flow contraction pressure loss to predict backpressure across an FTM?
  - How to discriminate between different PF models from literature?
  - Should developing flow be included in PF models?

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Methods / Techniques

How are we going to answer the questions

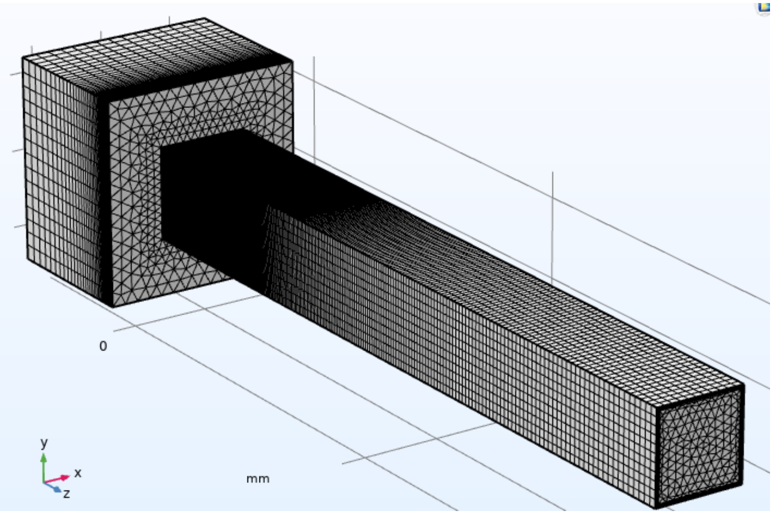
- Use three methods:
  1) CFD simulations
  2) Backpressure measurements on monolith cut to different lengths
  3) Backpressure measurements on particulate filters cut to different lengths

- Varying length of part helps with separating different contributions to backpressure
  - Although more complicated with filters than monolith

CFD Simulations

CFD Simulations

- Simulate flow in square channel with upstream flow contraction
  - Model for flow encountered in flow-through monolith
  - Investigate the effect of:
    - Gas velocity
    - Contraction ratio



- Enables us to:
  - Verify pressure change due to flow contraction at part entrance
  - Look at the impact of developing flow
  - Consider how upstream contraction affects developing flow in channel
    - Friction factor correlations typically assume flat velocity profile at channel entrance
    - But this does not exist when there is an upstream flow contraction

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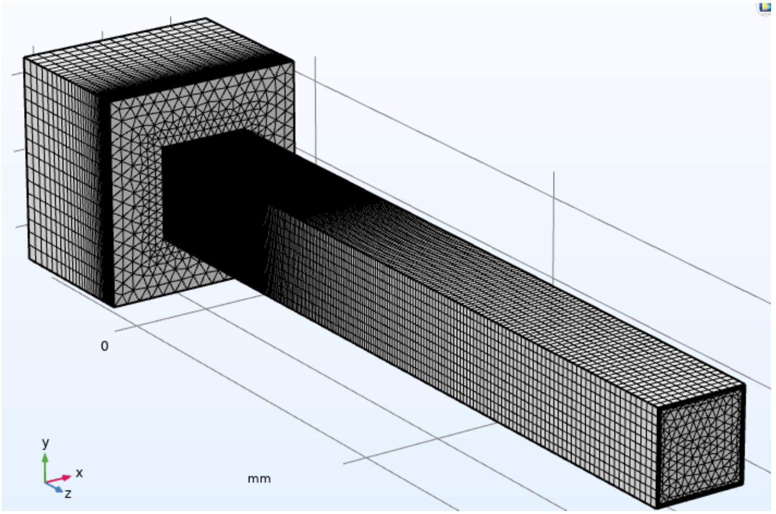
Simulations

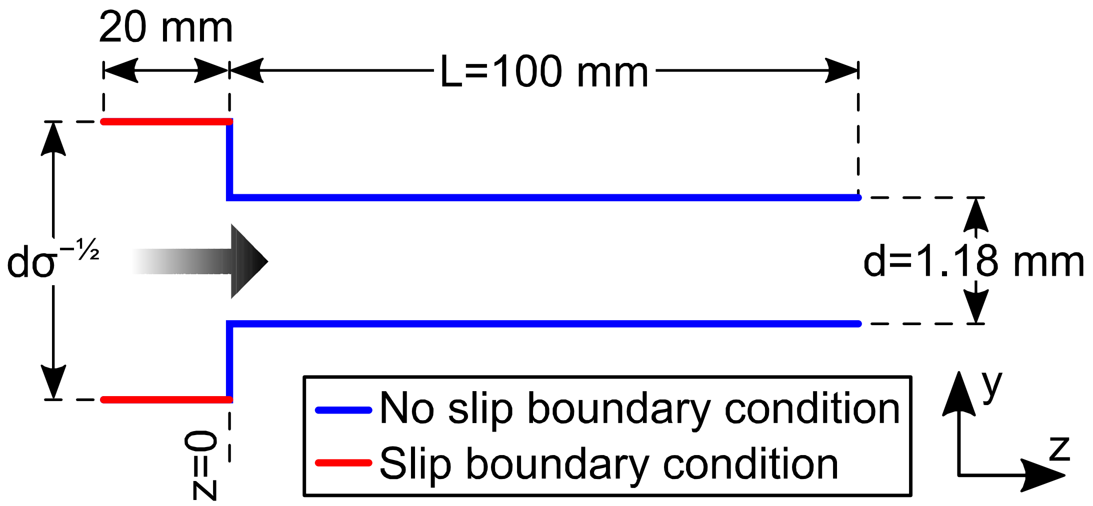


- CFD simulations run using Comsol Multiphysics®
  - Geometry consists of square channel with upstream contraction
  - Simulations run for:
    - Two flow rates
    - 4 contraction ratios, plus the case without an upstream contraction

Contraction ratio,

$$\sigma = \frac{\mathit{Area\ for\ flow\ after\ contraction}}{\mathit{Area\ for\ flow\ before\ contraction}}$$

Simulation Geometry:



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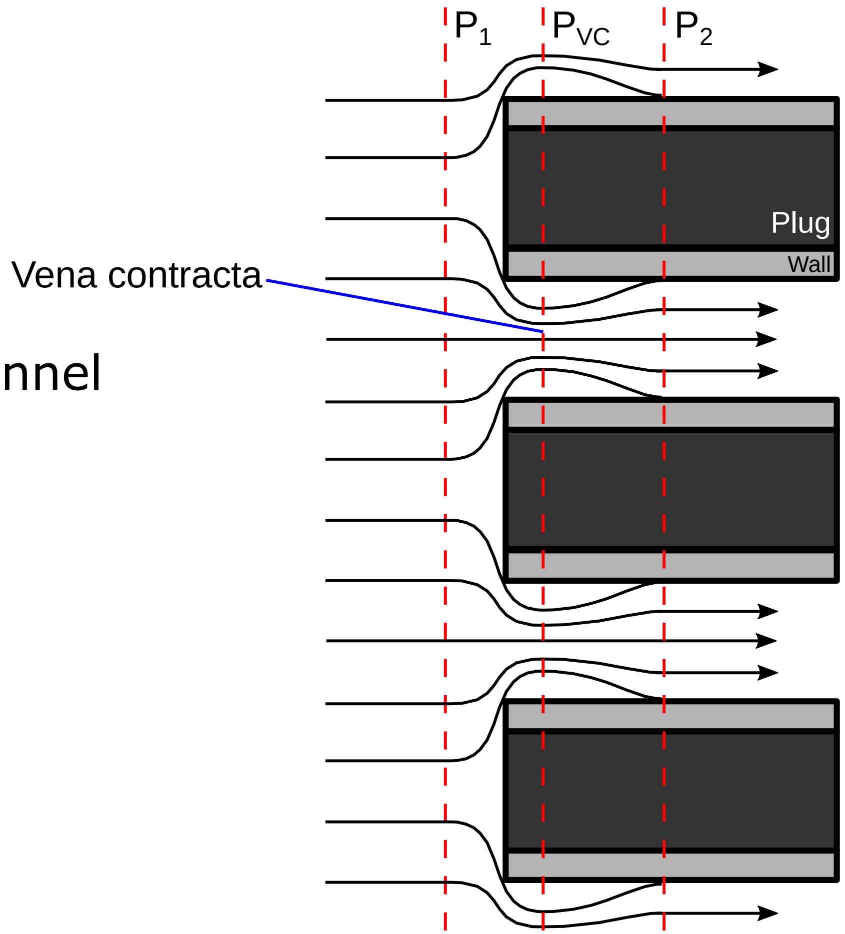
Pressure Drop Due to Flow Contraction

- During flow contraction at monolith/filter entrance:
  - Flow contracts down to vena contracta, narrower than channel
  - Then flow expands to fill channel
- Derive equation for pressure change assuming [2,5]:
  - Conservation of mechanical energy to vena contracta
  - Conservation of momentum over expansion
  - $\Delta P_{entr} = \left[2\alpha_{i0} - \alpha_{e,up}\sigma_{i0}^2 - \frac{2}{C_{Ci}} + \frac{1}{C_{ci}^2}\right]\frac{\rho V_{i0}^2}{2}$
- α_{i0} is momentum flux correction factor
  - Serves to correct ρV^2 to give momentum flux along channel
  - $\alpha = \frac{\langle u_z|u_z|\rangle}{\langle u_z\rangle^2}$



C_C=Coefficient of contraction
V=Mean channel velocity
α=Momentum flux correction factor
α_e=Kinetic energy flux correction
ρ=Fluid density
σ_{i0}=Contraction ratio

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Handling Backpressure Contribution Due to Accelerating Gas Flow to Laminar Flow Profile

- One contribution to backpressure for gas entering channel is force required to accelerate gas flow in channel to laminar flow profile
- If:
  1) Assume flow in channel always fully developed
     - Include this contribution with contraction pressure loss by using α_{i0} for laminar flow
  2) Account for developing flow in channel
     - Use friction factor correlation that accounts for developing flow
       - This includes contribution for force required to accelerate gas to laminar flow profile
     - Equation for flow contraction should just take gas to flat velocity profile, i.e. $\alpha_{i0} = 1$

Some Maths…

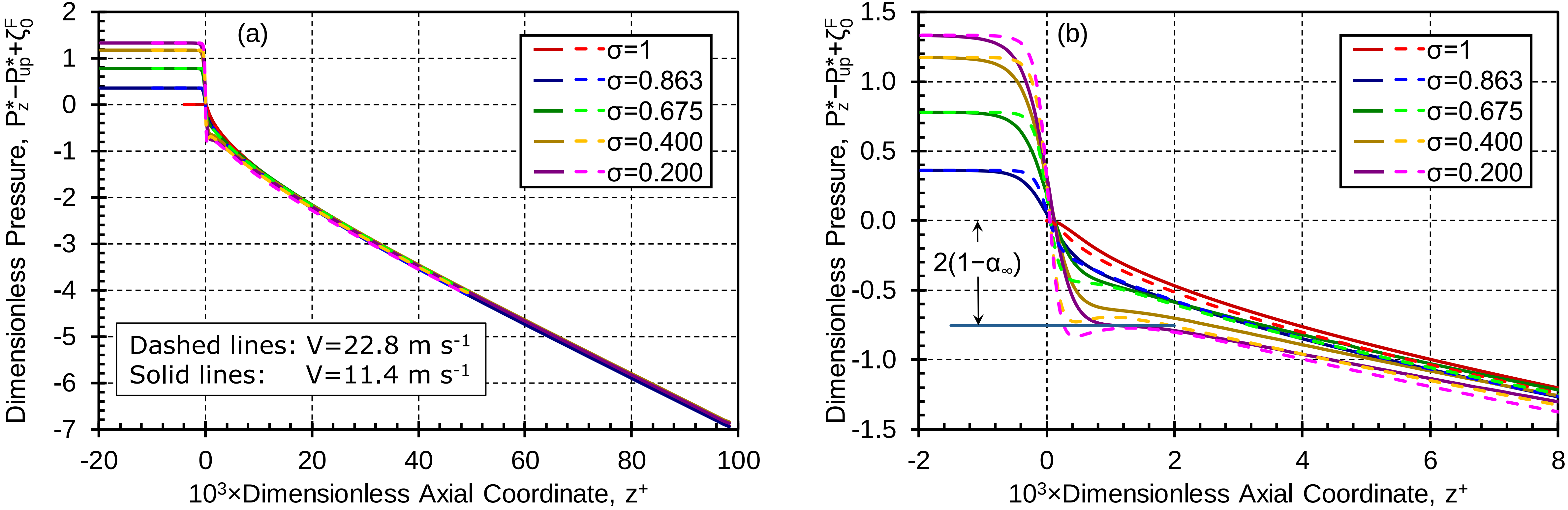
Dimensionless pressure and axial coordinate

- Pressure drop at distance z along channel, P_z, is given by:
  - $P_{up} - P_z = \left(\zeta_0^F + f_{app}\frac{4z}{d}\right)\frac{\rho V^2}{2}$
- Divide by ½ρV^2 and rearrange gives:
  - $P_z^* - P_{up}^* + \zeta_0^F = -4f_{app}Rez^+$
- From this equation, if f_{app}Re is a function of z^+ only, then a plot of $P_z^* - P_{up}^* + \zeta_0^F$ versus z^+ will always fall on the same line for 0≤z≤L irrespective of contraction ratio or flow rate
  - Not entirely true close to the contraction, but works otherwise

P_{up}=upstream pressure; P_z=pressure at z; z=axial coordinate; ζ_0^F=pressure drop due to contraction, taking fluid to a flat velocity profile; f_{app}=apparent friction factor; d=width of channel; ρ=fluid density; V=mean gas velocity in channel; $P^*=P/½\rho V^2$=dimensionless pressure; Re=dρV/μ=Reynolds number; z^+=z/dRe=dimensionless axial coordinate

Offset Dimensionless Pressure Versus Dimensionless Coordinate

All results line up when plotted this way, once away from the contraction



- Differences in pressure profile around the channel entrance due to contraction
- But once away from the contraction, all results fall on same line

Conclusions from Dimensionless Pressure Drop Plot

Consequences for backpressure prediction in flow-through monoliths

- If nondimensionalise pressure and axial coordinate and offset by pressure change due to flow contraction, all results fall on same curve away from contraction
  - For 2 flow rates and 5 contraction ratios (including no contraction)

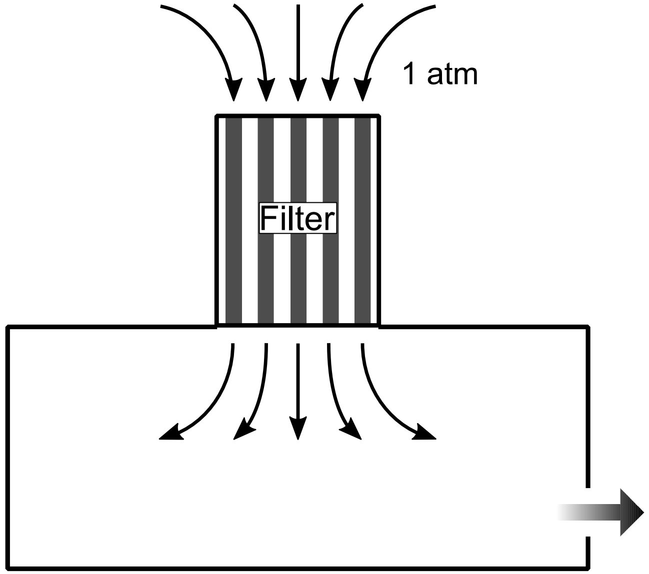
- From this conclude:
  - Equation we are using for pressure drop due to contraction is correct (equation validated ☺)
  - Can predict pressure drop by combining correlation for developing flow starting with a flat velocity profile with an equation for pressure drop for contraction taking the flow to a flat velocity profile
  - There is no need to include a viscous contribution to contraction pressure drop
    - At least for the (relatively high) flow rates considered
    - Unlike what Cornejo et al. found [3,4]
  - While contraction affects pressure close to contraction, overall pressure drop unaffected
    - Unlike what Cornejo et al. found [3,4]

Flow-Through Monolith Backpressure

Backpressure Measurements on Flow-Through Monolith

Effect of varying monolith length

- Measure backpressure of uncoated monolith, then progressively shorten part and re-measure backpressure
  - Gives backpressure as a function of length
    - Allows separation of along-channel pressure drop and pressure drop due to flow contraction into the part
  - As inlet pressure fixed (atmospheric) and exit pressure change zero (infinite expansion), equivalent to measuring pressure along channel
- Use SuperFlow® for measurements (cold flow)
- 400/3.5, 4.66 in diameter monolith



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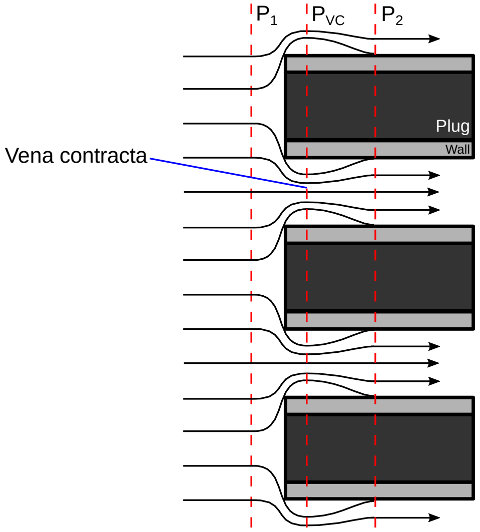
Backpressure Prediction Equations

- Assume incompressible flow
- Along channel pressure drop:
  - $\Delta P_{FTM} = \frac{F\mu\phi L}{d^2\rho}$



- Pressure drop due to flow contraction at entrance:
  - $\Delta P_{entr} = \left[2\alpha_{i0} - \alpha_{e,up}\sigma_{i0}^2 - \frac{2}{C_{Ci}} + \frac{1}{C_{ci}^2}\right]\frac{\rho V_{i0}^2}{2}$
  - Depending on value of α_{i0}, can take flow to flat profile or developed laminar flow profile
- Pressure change due to flow expansion at exit:
  - $\Delta P_{exit} = \sigma_{oL}(\alpha_{dwn}\sigma_{oL} - \alpha_{oL})\rho V_{oL}^2$

d=Hydraulic diameter of channel
F=Viscous loss coefficient
L=length of monolith
V=Mean gas velocity
α=Momentum flux correction factor
α_e=Kinetic energy flux correction
μ=viscosity
ρ=Fluid density
σ_{i0}=Contraction ratio
σ_{oL}=Expansion ratio
ϕ=Mean mass flux in channel

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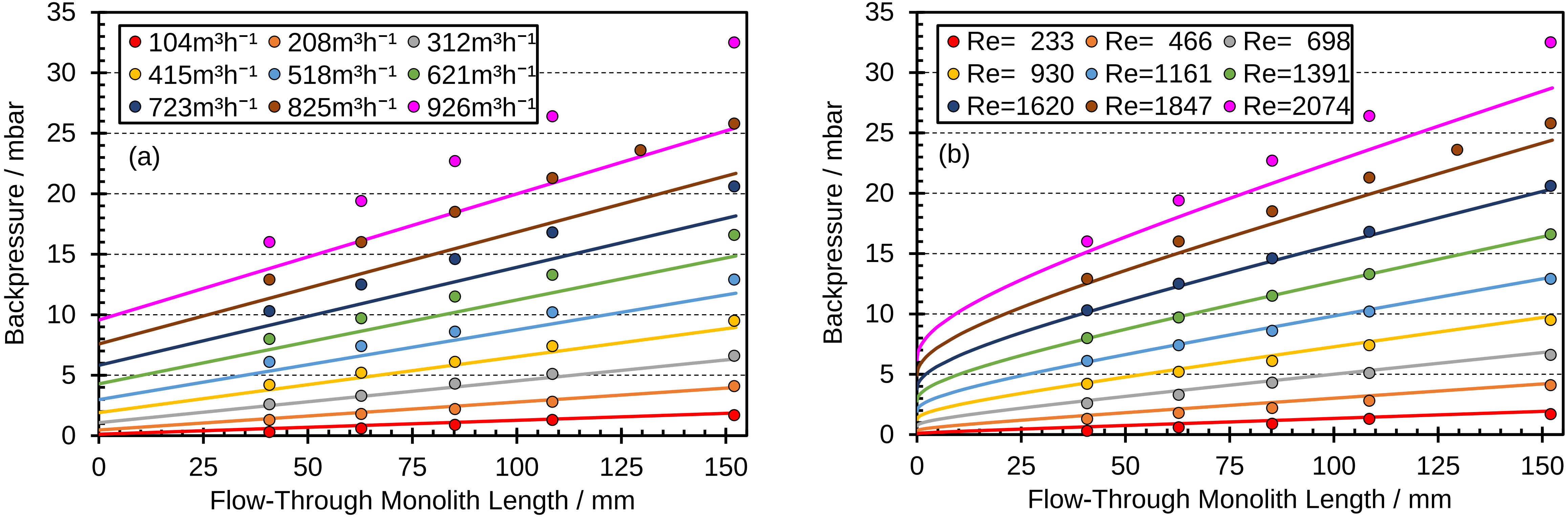
Contraction & expansion equations from [2,5]

Points: Measured data
Lines: Model prediction

Effect of Length on Monolith Backpressure

Model predictions for always-developed flow (left) & accounting for developing flow (right)



- Validates equation for flow contraction into monolith
- At lower flow rates (Re≤700), can assume flow always fully developed (left plot)
- Use of Gundlapally & Balokotiah [6] developing flow correlation improves prediction at higher flow
  - Model under predicts at very high flows (right plot)

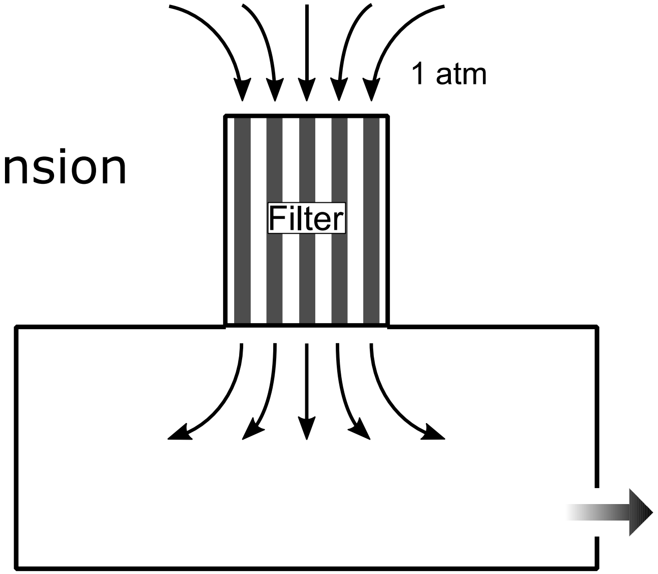
Summary: Cut-Monolith Backpressure Data

- Backpressure of monolith measured as function of length by progressively shortening the part
- Allows backpressure contributions of along-channel pressure drop and entrance & exit effects to be separated
  - Improved model validation
- At low flow rates, can assume flow always developed, but at higher flows need to include the effect of developing flow
- Gundlapally & Balakotaiah friction factor correlation for developing flow works well

Particulate Filter Backpressure Prediction

Validation of Particulate Filter Backpressure Prediction

- Measure backpressure of uncoated filter, then progressively shorten part and re-measure backpressure
    - After first cut have partial filter – by testing in both orientations double available data
- Full and partial filters are governed by the same differential equations
    - Just with different boundary conditions
    - So can use data also to validate model for full filter
- Use SuperFlow® for measurements (cold flow)
    - Expansion pressure change zero as effectively infinite expansion
- 292/10, 5.66×10in SiC filter



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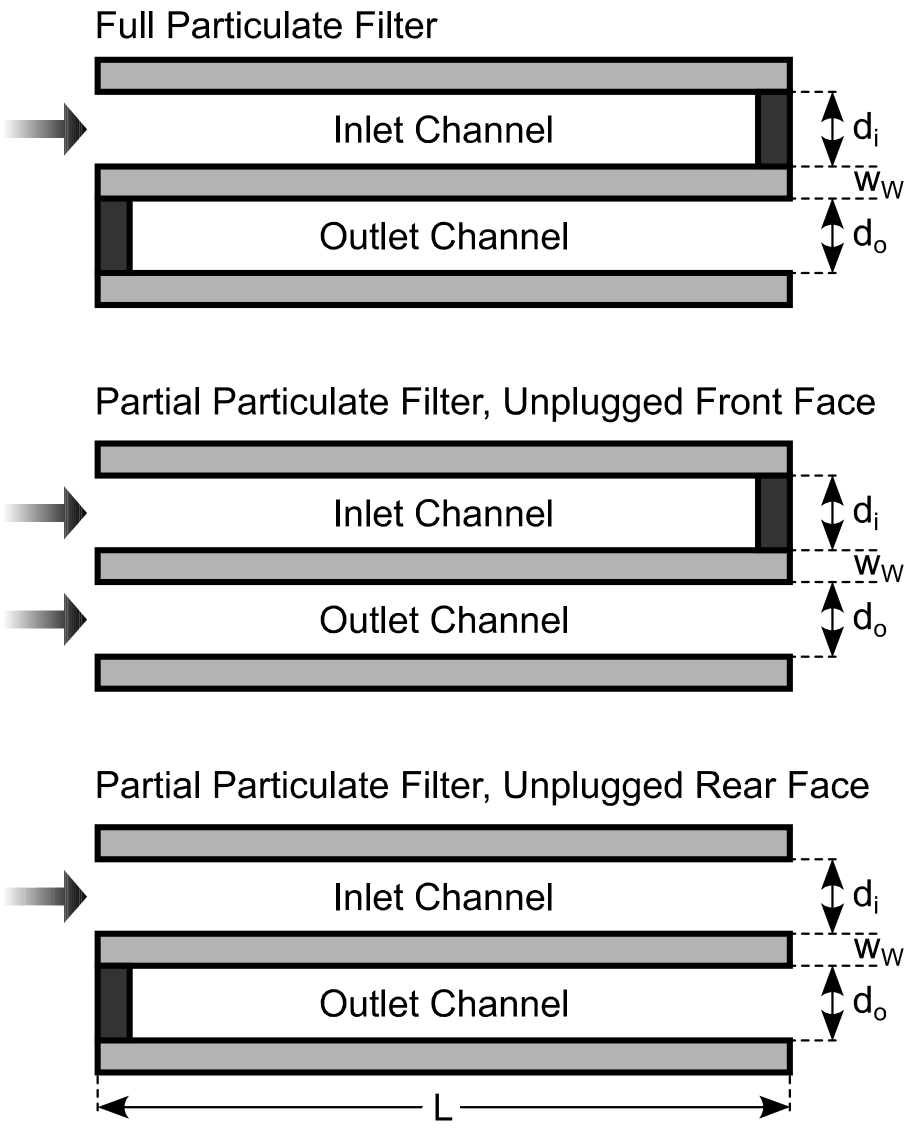
Full and Partial Particulate Filters: Nomenclature



- Label channels “inlet” or “outlet” according to the situation if the “missing” plugs were replaced

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Filter Backpressure Models

Will compare the prediction of four different filter models

d_i : Width of inlet channel
d_o : Width of outlet channel
F_i, F_o: Viscous loss coefficent
P_i, P_o: Pressure in inlet & outlet channels
z : Axial coordinate
α : Momentum flux correction factor
μ : Gas viscosity
ρ : Gas density
ϕ_i : Mass flux of gas along inlet channel
ϕ_o : Mass flux of gas along outlet chan

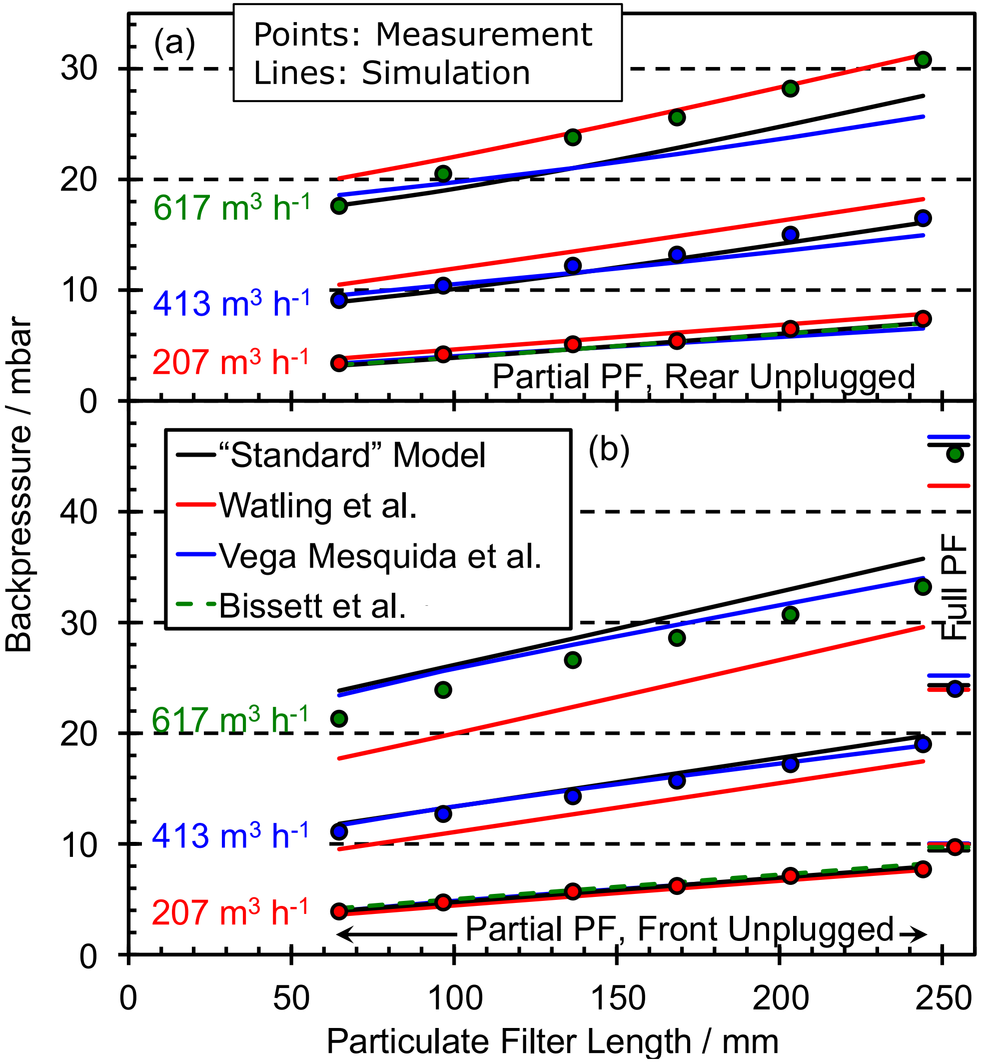
- Standard model:
  - $$\frac{\partial P_i}{\partial z} = -\frac{\alpha_i}{d_i^2}\frac{\partial d_i^2\phi_i^2/\rho_i}{\partial z} - \frac{F_i\mu_i\phi_i}{d_i^2\rho_i} \qquad \frac{\partial P_o}{\partial z} = -\alpha_o\frac{\partial \phi_o^2/\rho_o}{\partial z} - \frac{F_o\mu_o\phi_o}{d_o^2\rho_o}$$
  - Vega Mesquida el al. [7] model uses same equations but with different values for F & α
  - Bissett et al. [8] model uses same equation but with Re_W dependant F & α

- Model proposed in Watling et al., SAE 2017-01-0974 [2]:
  - $$\frac{\partial P_i}{\partial z} = -\frac{\alpha_i\phi_i}{d_i^2}\frac{\partial d_i^2\phi_i/\rho_i}{\partial z} - \frac{F_i\mu_i\phi_i}{d_i^2\rho_i} \qquad \frac{\partial P_o}{\partial z} = -\alpha_o\phi_o\frac{\partial \phi_o/\rho_o}{\partial z} - \frac{F_o\mu_o\phi_o}{d_o^2\rho_o}$$

Equations for pressure change due flow contraction & expansion extended to cover the unplugged faces of partial filters [1]

Comparison of Backpressure Prediction for Different Models

Permeability optimised for each model using data for two lowest flow rates



- All models look much the same at 207 m^3 h^{-1}
  - Watling et al. slightly different with unplugged rear
- Bisset et al. model only valid for $Re_W<3$ – only true for 207 m^3 h^{-1} for rear face unplugged
  - Hence, only show simulated data for 207 m^3 h^{-1}
- Poor prediction for Watling et al. model
  - Backpressure higher for unplugged front face
  - This model predicts the opposite
  - Therefore this model can be rejected
- Other models look much the same at 413 m^3 h^{-1}
- No model gives a good prediction at 617 m^3 h^{-1}
  - Perhaps assuming developed laminar flow incorrect

JM

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Modelling Developing Flow in Filters

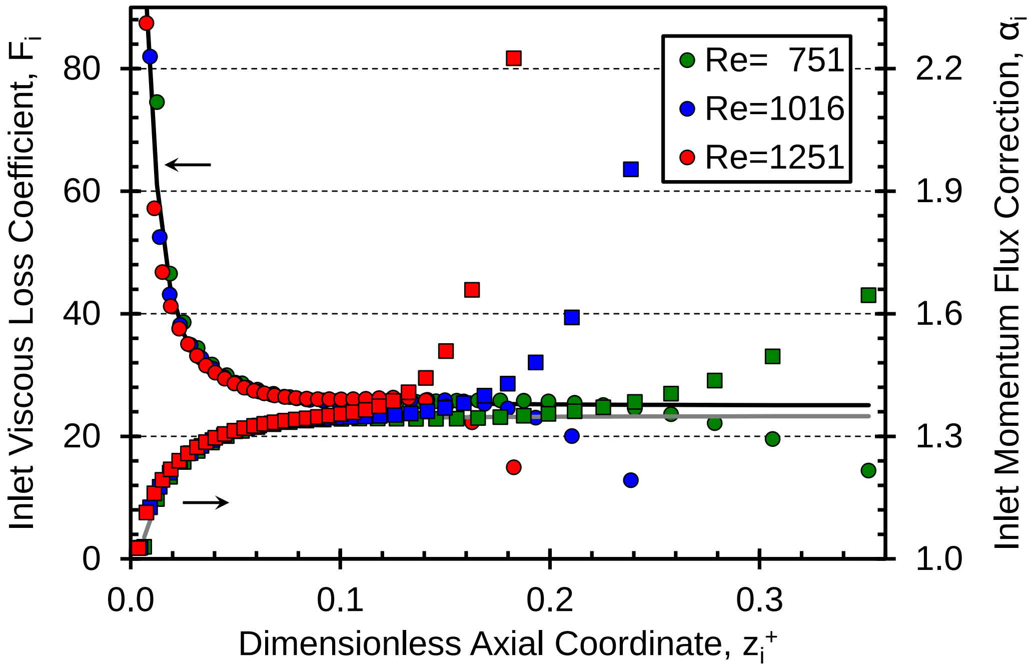
Defining a dimensionless distance for wall-flow filters

- Correlations for developing flow for monoliths in terms of dimensionless distance, z^+:
  - $z^+ = \frac{z}{d\,Re}$
  - This definition doesn't work for filters as Re varies along channels

- For filters, define dimensionless distance as:
  - $z_{i,l}^+ = \sum_{j=1}^{l} \frac{z_j - z_{j-1}}{d_i Re_{i,j}}$ $\qquad$ $z_{o,l}^+ = \sum_{j=1}^{l} \frac{z_j - z_{j-1}}{d_o Re_{o,j}}$
  - Divide channel up into a series of axial elements. Use values for these elements in summation

Demonstration That Dimensionless Distance Definition Works

Use CFD results from Cooper et al. [9]



- Plot F_i and α_i calculated from CFD against z_i^+ for 3 values of Re
- Predicted points line up for front of filter, where impact developing flow important
  - No correlation at rear of part where wall flow is important

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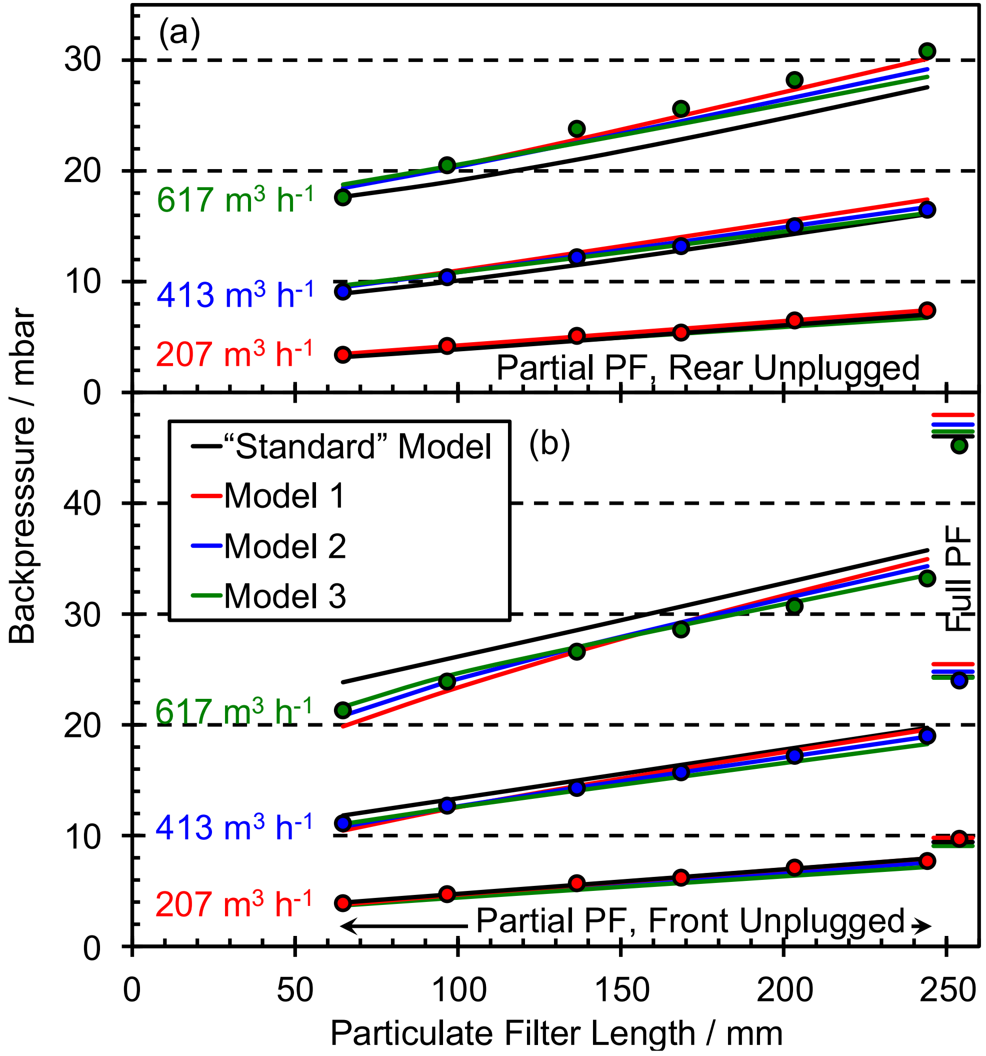
Models Including Developing Flow

Model 1	$\alpha_D = 1 + \frac{0.378}{1+0.004210(z^{+})^{-2}}$
Model 2	$\alpha_D = 1 + \frac{0.378}{1+0.001487(z^{+})^{-2}}$, $F_i = F_o = 26.613$
Model 3	$\alpha_D = 1 + \frac{0.378}{1+0.0005391(z^{+})^{-2}}$, $F_D = 27.779 - \frac{3.216}{1+0.0001720(z^{+})^{-2}}$

- Equations for F and α as functions of dimensionless distance
- This only applies to channels unplugged at the front face
- Form of equations taken from monolith friction factor correlation of Shah & London [10]

Comparison of Models Including Developing Flow



- All models give much the same prediction for the two lower flows
  - Expected as impact of developing flow only significant at higher flow
- Model including developing flow show some advantage over "standard" model, but none is perfect
  - More work required
  - Perhaps there is a limit to the range of flow rates at which a 1-dimension model would work

All models use same permeability

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Summary: Cut Filter Backpressure Data

- Generate data for validating filter model by measuring backpressure after repeatedly shortening part
  - After first cut, have partial filter
  - Test part in both orientation – backpressure higher when front face unplugged

- As full and partial filters governed by same differential equations, can use data for partial filter to validate model for full filter
- Method enables model discrimination

- None of models tried works really well at really high flow
  - Have tried models including developing flow
    - This improves prediction, but prediction is far from perfect

Conclusions

Conclusions

- A number of aspects of backpressure prediction for Flow-Through Monoliths (FTM) & Filters (PF) investigated:
  - Equation for pressure drop due to contraction validated against CFD and experimental data
  - Good backpressure prediction for FTM by combining equation for contraction to flat velocity profile and developing flow friction factor correlation assuming flat velocity profile at channel entrance
  - Backpressure data for partial PFs can be used to validate balance equations for full PF and to discriminate between models
  - Progress made on including developing flow in PF model; more work required

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